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Determining the End Points of the Score Confidence Interval Using Computer Program

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Authors' contrubutions

This work was carried out in collaboration between both authors. Author JI designed the study, performed the statistical analysis, wrote the protocal and wrote the first draft of the manuscript. Author RRS managed the analyses of the study and managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

For interval estimation of a proportion the Score Interval is quite accurate. It has good reviews in the Statistics literature. But the problem is that it is not used enough. A reason is that many consider it is complicated. In this paper, we suggest a program and other things that we hope will make the Score Interval more suitable to use in the field of statistics.

Keywords: Confidence interval; score interval; binomial parameter; Wald procedure.

1 Introduction

A basic analysis in statistical inference is constructing a confidence interval for a bi nomial parameter P [1,2, 3,4,5]. The simplest interval which is almost universally used is,

$$\hat{P} \pm z_{\frac{\alpha}{2}} \left[\frac{\hat{P}(1-\hat{P})}{n}\right]^{\frac{1}{2}} \tag{1.1}$$

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where $\hat{P}$ is the sample proportion, n is the sample size and $z_{\frac{\alpha}{2}}$ denotes the $1-\frac{\alpha}{2}$ quantile of the standard normal distribution. For instance α = 0.05 for a 95 % confidence interval, $\alpha = 0.10$ for a 90% confidence interval, etc. This interval is derived from the Wald large sample confidence interval and is commonly referred to as the Wald interval [6].

So it seems at first glance that the problem is simple and has a clear solution. Actually the problem is a difficult one with several complexities. It is widely recognized that Wald interval coverage probability is poor for P near 0 or 1. It is known that the Wald interval performs poorly unless n is large [7]. Most statistics books take this into account by requiring that this interval should be used only when $\min(np, n(1-P))$ is at least 5 or 10 [8].

A considerable literature exists about this and other less common methods for constructing a confidence interval for P [9,10,11,12]. Santner and Duffy [13] and Vollset [4] reviewed a variety of methods. One of the methods is the Clopper-Pearson “exact" interval [14]. This method is widely used and has the advantage of a coverage probability of at least $1-\alpha$ for every possible value of P. The Score method [15] discussed by Agresti and Coull [16], is arguably the best procedure for constructing a confidence interval for a population proportion. Guan [17] introduced the generalized score method which computes easily and reduces the spike fluctuations of the score method. Also Bayesian methods are effective for constructing confidence intervals for a population proportion. In addition other effective procedures such as the Arcsin, Logit and Jeffres prior intervals are discussed in Brown, Cai, Das Gupta [8]. The Jeffres prior interval is a special case of a Bayes procedure with a non-informative prior. Bayes procedures with a non- informative prior have a good track record in constructing confidence intervals for P; see Wasserman [18]. Wang [19] discusses methods for constructing the smallest exact confidence intervals. Price and Bonnet and Zao, Huang and Zhang [20] use the Score interval to construct a confidence interval for a linear function of binomial proportions.

However, most effective procedures are too complicated to use much in Statistics. Therefore Agresti and Coull [16] introduced the Adjusted Wald (AC) procedure. The AC method consists of adding two successes and two failures to the data and then proceding as in the Wald interval. This method is simple, easy to use and accurate. The accuracy of the AC procedure is due to its midpoint and width being almost the same as those of the Score procedure for a 95% confidence interval. Actually the AC interval is a simplified version of the Score interval.

At the present time, the Wald interval is almost exclusively used in everyday practical statistics. Some reasons for its popularity are that it is easy to motivate and easy to use. Under the right conditions such as $np(1-P) \geq 10$, it is reasonably accurate.

This article is focused on the Score confidence inteval procedure. The emphasis is on confidence interval procedures that have a “closed form” [8,21]. By “closed form” we mean there is a simple formula to compute the end points. The Score confidence interval is arguably the most accurate of the closed form confidence interval procedures. We hope this article will make the Score interval more popular.

2 The Score Interval

The Wald confidence interval for a population P is by far the most commonly used confidence interval for a population proportion. The end points P of this interval are:

$$\hat{P} \pm z\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} = P \tag{2.1}$$

Here $\hat{P}$ is the sample proportion, n is the sample size and z is the z-value for the confidence level. That is, z = 1.960 for a 95 % confidence level, z = 1.645 for a 90 % confidence level, etc.

A starting point for the Score interval is the Wald Interval (2.1). If we use equatioin (2.1) and change the $\hat{P}$'s under the radical to P's, we have Score interval (2.2)

$$\hat{P} \pm z\sqrt{\frac{P(1-P)}{n}} = P \quad (2.2)$$

However, it is expedient to rewrite (2.2) as equation (2.3).

$$\hat{P} - P = \pm z\sqrt{\frac{P(1-P)}{n}} \quad (2.3)$$

If we square both sides of (2.3) we end up with a very complicated quadratic equation in P. If we solve this equation, the result is

$$C\hat{P} - (1-C)(0.5) \pm z\sqrt{\frac{C\hat{P}(1-\hat{P}) + \frac{(1-C)}{4}}{n+z^2}} \quad (2.4)$$

Here, $C = \frac{n}{n+z^2}$ and $1 - C = \frac{z^2}{n+z^2}$

Equation (2.4) shows that the mid-point of the Score interval is a weighted average of $\hat{P}$ and $\frac{1}{2}$, and that the variance is the same weighed average of $\hat{P}(1-\hat{P})$ and $\frac{1}{4}$, but uses $n + z^2$ in place of n.

Equation (2.4) can be simplified to

$$\frac{x+\frac{z^2}{2}}{n+z^2} \pm \frac{z}{n+z^2}\sqrt{xq + \frac{z^2}{4}} \quad (2.5)$$

Here, $q = 1 - \hat{P}$ and x is the number of successes in the sample.

We can use (2.5) to compute the end points of the Score Confidence interval. Alteranately, set

$$A = \frac{x+\frac{z^2}{2}}{n+z^2},\ B = \frac{z}{n+z^2}\sqrt{xq + \frac{z^2}{4}}$$

Then, the Score Interval = $(A - B, A + B)$ (2.6)

The Score confidence interval can be made more useful if a programmable calculator is available. Many who use Statistics have a programmable calculator such as the TI-83 or TI-84. Therefore we have included a TI-83 program to find the end points of the Score confidence interval. The same program with minor changes can be used in an R program, Maple software and in Microsoft excel. Basically, the program accepts X, N and C as input and computes Z, D, E and F from the input, where

$$E = x + \frac{z^2}{2},\ D = \sqrt{xq + \frac{z^2}{4}},\ \text{and } F = n + z^2$$

Then

$$\frac{E-D}{F} \quad and \quad \frac{E+D}{F} \quad (2.7)$$

are the end points of the score interval.

The Program: Score Interval

: Prompt X, N

: Input “C-level?", C

:$InvNorm\left(\frac{C}{2}+0.5\right) \to Z$

: $\left(\frac{Z^2}{2}\right) \to E$

: $\left(\frac{1-X}{N}\right) \to Q$

: $Z\sqrt{\left(\frac{Z^2}{4}+XQ\right)} \to D$

: $(N+Z^2) \to F$

:$\left(\frac{E+D}{F}\right) \to R$

: $\left(\frac{E-D}{F}\right) \to L$

: $round(R,4) \to R$

: $round(L,4) \to L$

: Disp “END PTS", E, F

This program prompts a user for X, the number of successes, N, the sample size and C-Level for the confidence level.

An example is

x: 4(press enter)

N: 10(press enter)

$C - level$ 0.95 (press enter)

The output is

END PTS

0.1682

0.6873

Also, one could use

$E = x + \frac{z^2}{2}$, $D = \sqrt{xq + \frac{z^2}{4}}$, and $F = n + z^2$

The Score Interval is $\left(\frac{E-D}{F}, \frac{E+D}{F}\right)$.

3 Accuracy of the Score Confidence Interval

It is well known that the Score procedure is quite accurate. However it may be useful to provide some evidence of this accuracy. It would be nice if there was a 90 % confidence interval for a population proportion *P* that covered each value of *P* with a probability of 0.90. But no such procedure exists. The most one can expect is that the average coverage of different values of *P* is close to 0.90. The Score confidence interval is excellence in this regard. The Wald interval has poor average coverage except when n is large and *P* not close to zero or one. A good source of the accuracy of the Score confidence interval is [16,22,23]. We summaize their results in the following Table 1.

The following Table 1 shows that the average coverage of the Score interval is very good when compared to other procedures.

Table 1. The Mean coverage probabilites of Nominal 95% confidence intervals for the Binomial parameter *P* for different sample sizes with Root Mean Square Error in the parenthesis

Sample size	n = 5	n = 15	n = 50
Score confidence interval	0.955 (0.029)	0.953 (0.019)	0.952 (0.012)
Exact confidence interval	0.990 (0.041)	0.980 (0.031)	0.969 (0.022)
Wald confidence interval	0.641 (0.400)	0.819 (0.238)	0.901(0.133)
Wald confidence interval with t	0.664 (0.391)	0.837(0.233)	0.905 (0.131)
Continuity-Corrected Score (Con-Cor Score)	0.987 (0.329)	0.979 (0.030)	0.969 (0.021)

Note that for different sample sizes the average coverage for the Score interval is closer to 0.95 in all cases. Also, the Root Mean Square Error is lower for the Score interval than that of any other procedure in all cases. We point out that the exact and the continuity-corrected score (Con-cor Score) are not closed form procedures.

4 Discussion

Experience has shown that a complicated confidence interval procedure will not be used much unless there is computer program that the user only has to input n (sample size), x (number of success) and C (confidence level). Thus we have included a short TI-83 program to compute the end points of the Score interval. Alternately we have also included other formulas (2.6) and (2.7) that could be of interest to some.

It would be nice if there was a 90 % confidence interval procedure for a population proportion *P* that covered each value of *P* with a probability of 0.90. But no such procedure exists. The most one can expect is that the average coverage of different values of *P* is close to 0.90. The score confidence interval is excellent in this regard. The Wald interval has poor average coverage except when n is large and *P* not close to zero or one. The score procedure has excellent properties for all values of n, *P* and confidence levels.

5 Conclusion

It is well known that the Score interval procedure works well for all sample sizes, all confidence levels and all possible number of success in the sample. But the problem is that the Score confidence interval procedure is not used much. The main purpose of this paper is to make the Score interval easier to use by introducing a TI-83 program and other formulas. Also, it would be nice if Texas Instruments or some other company puts in a procedure to compute the end points of the Score program the same way it does for the Wald confidence interval for a population proportion.

Competing Interests

Authors have declared that no competing interests exist.

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